

1. By considering infinitesimal rotations and boosts about the appropriate axes in spacetime construct the rotation generators, L_i and the boost generators, K_i . Find the commutation relations between these generators. You should consider pure boosts and rotations separately about the x, y, z axes and by looking at the infinitesimal transformations, find the corresponding generators. Show that the commutation relations at the top of page 6 of lecture 3 notes are satisfied. You may check only one from each of the three.
2. (a) Consider two events that are separated by a space-like interval. Show that there exists a Lorentz frame in which they are simultaneous but there is no Lorentz frame in which they occur at the same spatial point.
 (b) Consider two events that are separated by a time-like interval. Show that there exists a Lorentz frame in which they occur at the same spatial point but there is no Lorentz frame in which they are simultaneous.
3. Consider a conserved current j^μ , i.e., $\partial_\mu j^\mu = 0$. The total charge is defined by,

$$Q = \int d^3x j^0(\vec{x}, t).$$

- (a) Show that Q is time independent, i.e., $dQ/dt = 0$
- (b) Show that Q transforms as a scalar under Lorentz transformations, i.e., if

$$Q' = \int d^3x' j'^0(\vec{x}', t')$$

is the charge defined with respect to a Lorentz transformed coordinate system $x' = \Lambda x$, then you need to show that $Q' = Q$.

(You may want to expand on the proof given in Weinberg's book "Gravitation and Cosmology" in chapter 2 (special relativity) in the section on "charges and currents".)